Panel Data, Factor Models, and the Solow Residual*

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Abstract

In this paper we discuss the Solow residual (Solow, 1957) and how it has been interpreted and measured in the neoclassical production literature and in the complementary literature on productive efficiency. We point out why panel data are needed to measure productive efficiency and innovation and thus link the two strands of literatures. We provide a discussion on the various estimators used in the two literatures, focusing on one class of estimators in particular, the factor model. We evaluate in finite samples the performance of a particular factor model, the model of Kneip, Sickles, and Song (2009), in identifying productive efficiencies. We also point out that the measurement of the two main sources of productivity growth, technical change and technical efficiency change, may be not be feasible in many empirical settings and that alternative survey based approaches offer advantages that have yet to be exploited in the productivity accounting literature.

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1 Introduction

In this paper we discuss the Solow residual (Solow, 1957) and how it has been interpreted and measured in the neoclassical production literature and in the complementary literature on productive efficiency. We point out why panel data are needed to measure productive efficiency and innovation and thus link the two strands of literatures. We provide a discussion on the various estimators used in the two literatures, focusing on one class of estimators in particular, the factor model. We evaluate in finite samples the performance of a particular factor model, the model of Kneip, Sickles, and Song (2009), in identifying productive efficiencies. We also point out that the measurement of the two main sources of productivity growth, technical change and technical efficiency change, may be not be feasible in many empirical settings and that alternative survey based approaches offer advantages that have yet to be exploited in the productivity accounting literature.

The plan of the paper is as follows. In the next section we discuss how productivity growth has been measured and how certain aspects of its evolution have been disregarded by classical economic modeling that abstracted from the realities of inefficiency in the production process. We also point out how closely linked technical change and technical efficiency change can appear and how it is often difficult to discern their differences in productivity growth decompositions. Section 3 discusses alternative survey based methods that may be implemented to assess the contributions of technical innovation and technical efficiency change to productivity growth through the development of a series of "Blue-chip" consensus country surveys that could be collected over time and which could serve as a new measurement data source to evaluate governmental industrial and competition policies. Section 4 outlines methods that have been proposed to measure productivity, efficiency, and technical change as well as focusing on the class of factor models which may have an advantage over other methods proposed to identify productive efficiencies. Section 5 focuses on one such factor model developed by Kneip, Sickles, and Song (2009) for generic stochastic process panel models and which we reparametrize to estimated time-varying and firm-specific efficiency while allowing a common-stochastic trend to represent technical change. Concluding remarks are provided in section 6.

2 Productivity Growth and Its Measurement

Productivity growth is the main determinant of changes in our standard of living. Although anecdotal evidence about particular levels of wealth creation is interesting it does not provide governments, sectors, or individual firms with an adequate picture of whether growth in living standards is economically significant and how the growth in living standards is distributed, both within countries and among countries. The linkages between productivity growth and living standards is clearly seen during different epochs for the U. S. economy in Figure 1 (Koenig, 2000). Growth in GDP per capita tends to rise and fall in conjunction with growth in labor productivity.

<<Figure 1 about here>>

2.1 Classical Residual based Partial and Total Factor Productivity Measurement

Measurements of productivity usually rely on a ratio of some function of outputs (Y_i) to some function of inputs (X_i). To account for changing input mixes, modern index number analyses use some measure of total factor productivity (TFP). In its simplest form, this is a ratio of output to a weighted sum of inputs:

$$TFP = \frac{Y}{\sum a_i X_i}. \tag{1}$$

Historically, there are two common ways of assigning weights for this index. They are to use either an arithmetic or geometric weighted average of inputs: *the arithmetic weighted average*, due to Kendrick (1961), uses input prices as the weights; *the geometric weighted average* of the inputs, attributable to Solow (1957), uses input expenditure shares as the weights. some reference point to be useful. **Solow's measure** is based on the Cobb-Douglas production function with constant returns to scale, $Y = AX_L^{\alpha} X_K^{1-\alpha}$ and leads to the TFP measure:

$$TFP = \frac{Y}{X_L^{\alpha} X_K^{1-\alpha}}. \tag{2}$$

At cost minimizing levels of inputs, the α parameter describes the input expenditure share for labor. The TFP growth rate would be described by: $T\dot{F}P = \frac{dY}{Y} - \left[\alpha \frac{dX_L}{X_L} + (1-\alpha)\frac{dX_K}{X_K}\right]$. In applied work, both sets of weights (Kendrick's and Solow's) are often inconsistent with the observed data.

Endogenous growth models were developed to weaken the strong neoclassical assumption that long-run productivity growth could only be explained by an exogenously driven change in technology and that technical change was exogenous. The classic model put forth by Romer (1986), which began the "new growth theory," allowed for non-diminishing returns to capital due to external effects. For example, research and development by a firm could spill over and affect the stock of knowledge available to all firms. In the simple Romer model firms face constant returns to scale to all private inputs. The production function frontier is formulated as

$$Y = A(R)f(K, L, R). \tag{3}$$

In the "new" growth theory, the production frontier is shifted by factors $A(R)$ where R is the stock of some privately provided input R (such as knowledge) that is endogenously determined. What is its source? Arrow (1962) emphasized "learning-by-doing." Recently, Blazer and Sickles (2010) have pursued this as an alternative to the stochastic frontier model. Romer (1986) modeled A as a function of the stock of research and development. Lucas (1988) modeled A as a function of stock of human capital.

Where multiple outputs exist, TFP can also be described as a ratio of an index number describing aggregate output levels(y_j) divided by an index number describing aggregate input levels(x_i). As such, they derive many of their properties based the assumptions of the underlying aggregator functions used. Fisher (1927) laid out a number of desirable properties for these index numbers. Many of these properties are easily achievable, while others are not. Following Jorgenson and Griliches (1972), a (logarithmic) total factor productivity index can be constructed as the difference between log output and log input indices, i. e.

$$\ln TFP = \ln y_t^1 - \ln x_t^1. \tag{4}$$

An implication of the endogenous growth model is that if a time trend is added to the standard neoclassical production function then the trend must be stochastic. This clearly has implications for stationarity (Reikard, 2005). Recent work by Kneip, Sickles, and Song (2009) has addressed the estimation issues that are associated with estimating the endogenous technical change in the presence of technical efficiency change.

2.2 Technical Efficiency in Production

It is often quite difficult to separate the impacts of technical change from constraints in the use of the existing technology, or technical efficiency. An example of the overlay of technology (and its change) and efficiency (and its change) can be found in the classic story of the reason "behind" the specifications of the solid rocket boosters (SRB's) for the space shuttle. The SRBs are made by Morton Thiokol at a factory in Utah. Originally, the engineers who designed the SRBs wanted to make them much fatter than they are. Unfortunately, the SRBs had to be shipped by train from the factory to the launch site in Florida and the railroad line runs through a tunnel in the mountains. The SRBs had to be made to fit through that tunnel. The width of that tunnel is just a little wider than the U.S. Standard Railroad Gauge (distance between the rails) of 4 feet, 8.5 inches. That is an odd number and begs the question of why that gauge was used? It was used because US railroads were designed and built by English expatriates who built them that way in England. The English engineers do so because the first rail lines of the 19th century were built by the same craftsmen who built the pre-railroad tramways, which used the gauge they used. The reason those craftsmen chose that gauge was because they used the same jigs and tools that were previously used for building wagons, and the wagons used that wheel spacing. The wagons used that odd wheel spacing since if the wagon makers and wheelwrights of the time tried to use any other spacing, the wheel ruts on some of the old, long distance roads would break the wagon axles. As a result, the wheel spacing of the wagons had to match the spacing of the wheel ruts worn into those ancient European roads. Those ancient roads were built by Imperial Rome for theirir legions and the roads have been used ever since. The initial ruts, which everyone else had to match for fear of destroying their wagons, were first made by Roman war chariots. Since the chariots were made by Imperial Roman chariot makers, they were all alike in the matter of wheel spacing. Why 4 feet, 8.5 inches? Because that was the width needed to accommodate the rear ends of two Imperial Roman war horses. Therefore, the railroad tunnel through which the late 20th century space shuttle SRBs must pass was excavated slightly wider than two 1st century horses' rear-ends and consequently, a major design feature of what is arguably the world's most advanced transportation system was specified by the width of the read-end of a horse.

This story is a bit of folk lore whimsy and has an oral and written tradition that is as old as the aging space shuttle fleet (see, for example, one of the many url's where it is documented at http://www.astrodigital.org/space/stshorse.html). Although this is just one of many anecdotes, it illustrates how constraints to adopting the most advanced technology may arise seemingly by a random process, in fact arise by historical precedent. We thus turn to an alternative to the Solow type neoclassical model of productivity and focus on a component neglected in the traditional neoclassical approach, technical inefficiency. Since the fundamental theoretical work by Debreu (1951), Farrell (1957), Shepherd (1970) and Afriat (1972), researchers have established a method to measure the intrinsically unobservable phenomena of efficiency. Aigner, Lovell, and Schmidt (1977), Battese and Cora (1977), and Meeusen and Van den Broeck (1977) provided the econometric methods for the applications waiting to happen. The linear programming methodology, whose implementation was made transparent by Charnes, Cooper, and Rhodes (1978), became available at about the same time. The U. S. and international emphasis on deregulation and the efficiencies accruing to increased international competition due to the movement to lower trade barriers provided a fertile research experiment for efficiency modelers and practitioners.

The efficiency score, as it is usually measured, is a residual. Parametric assumptions about the distribution of efficiency and its correlation structure often are made to sharpen the interpretation of the residual. However, that efficiency measurement should be highly leveraged by parametric assumptions is by no means a comforting resolution to this measurement problem. Productivity defined by the Solow residual is a reduced form concept, not one that can be given a structural interpretation. Different efficiency estimators differ on what identifying restrictions are imposed. Not surprisingly, different efficiency estimators often provide us with different cross-sectional and temporal decompositions of the Solow residual.[1] Kumbhakar and Lovell (2000) and Fried, Lovell, and Schmidt (2008) have excellent treatments of this literature. It addresses the continuing debate on how the distributional assumptions made in Pitt and Lee (1981), Kumbhakar (1990), Battese and Coelli (1992), and others drive the estimates of efficiency. The robust and efficient estimators have been developed by

[1] Since cross-sectional data are used, the efficiencies estimated are typically conditional expectations, as it is mentioned in Simar and Wilson (2010).

Park, Sickles, and Simar (1998, 2003, 2007), Adams, Berger, and Sickles (1999), Adams and Sickles (2007). These share a number of generic properties with the estimators proposed by Schmidt and Sickles (1984) and Cornwell, Schmidt, and Sickles (1990).

2.3 Difficulty in Measuring the Decomposition of Productivity Growth into Technical Change and Technical Efficiency Change

We point out below problems in decomposing productivity change into its innovation and its efficiency change components. One conclusion from this discussion is that it simply may not be possible from purely econometric models, no matter how sophisticated, to model structurally the role of innovation and the role of efficiency in determining TFP growth. We give two illustrations. The first is based on experience gleaned by Sickles as the Senior Research Coordinator for the Development Economic Policy Reform Analysis Project (DEPRA), USAID/Egyptian Ministry of Economy, Contract No. 263-0233-C-00-96-00001-00. A portion of this research was the basis for Getachew and Sickles (2007). The study analyzed the impact of regulatory and institutional distortions on the Egyptian private manufacturing sector from the mid 1980's to the mid 1990's. We focused on the impact of economic reforms undertaken since 1991. The second is based on work of Sickles and Streitwieser (1992, 1998) who addressed the impact of the Natural Gas Policy Act of 1978 on the U. S. interstate natural gas transmission industry.

2.3.1 How Can We Identify Specific Constraints at the Macro Level?

The Development Economic Policy Reform Analysis Project in Egypt was a USAID/World Bank project that began in the mid-1980's and lasted through the mid-1990's. The aim of the project was to transition from the planned economy left by the Soviet Union to a private sector market economy via a structural adjustment program. Initial efforts focused on macroeconomic stabilization which involved a reduction of the fiscal deficit through a variety of measures. These measures included:

1. cuts in public investment and subsidization programs;
2. tax reforms, particularly through the introduction of a general sales tax;
3. improvements in collection; and
4. monetary policy tightening to fight inflation.

The structural adjustment program also involved extensive price liberalization that affected each sector of the Egyptian economy. This involved:

1. adjustments of relative prices;

2. removal of all export quotas, except for tanned hide, in the trade and financial sectors;
3. lifting of tariffs on almost all imported capital goods;
4. removal of constraints on nominal interest rate ceilings, administrative credit allocation, foreign exchange controls and prohibitions against international capital mobility; and
5. reform of labor laws, which gave employers the right to hire and lay off workers in accordance with economic conditions.

How do we develop a model that identifies such a plethora of structural changes in the Egyptian economy? One approach was undertaken by Getachew and Sickles (2007) who utilized a virtual cost system and were able to identify allocative distortions that existed before the reforms were undertaken and those that existed after the reforms had worked their way through the Egyptian private sector after the deregulatory reforms. Getachew and Sickles found substantial welfare benefits accruing to the Egyptian economy due to these reforms in total. Unfortunately, the specific determinants of the benefits of market reforms could not be ascertained since the specific constraints could not be modeled and thus incorporated into an estimable structural model.

2.3.2 How Can We Identify Specific Constraints at the Micro Level?

Another illustration is found in the regulatory change accompanying the U.S. Interstate Natural Gas Policy Act of 1978. The regulatory history of natural gas transmission industry is long and complicated. Figure 2 provides us with a schematic diagram that outlines the maximum ceiling price schedules from 1978 to 1985 and the 24 different price combinations over the period for different categories of natural gas (for details, see Sickles and Streitwieser, 1992). As Figure 2 points out, the regulations and their impact on the various firms involved in the deregulatory initiatives are enourmously complex. A formal model of the constraints in an estimable structural econometric model is simply not feasible. One can clearly see the difficulties inherent in any attempt to parsimoniusly quantify the constraints, not to mention the difficulties one would have in ultimately interpreting how these constraints could impact optimal natural gas transmission decisions.

<<Figure 2 about here>>

3 Alternatives to Measurement of Technical Change and Technical Efficiency Change

3.1 Survey-Based Methods for Decomposing Total Factor Productivity Growth into Technical Change and Technical Efficiency Change: A New Blue Chip Indicator

Various approaches to decomposing total factor productivity into sources that are due to efficiency change and due to technological change have been discussed. One popular index number approach based on the decomposing the Malmquist index (Caves et al., 1982) was introduced by Färe et al. (1992). Of course, regression based approaches using either traditional neoclassical growth models, growth models in which endogenous growth is allowed, or growth models in which inefficiency is explicitly introduced via a frontier technology offer potentially richer empirical specifications and a more structural determination of the sources of productivity growth. However, all approaches suffer due to poor empirical proxies for the measures of loosening constraints to business activity. One possibility to circumvent the paucity of reliable empirical measures of the determinants of productivity growth would be to conduct a structured survey of business leaders, political leaders World Bank, International Monetary Fund, and Non-governmental Organizations to identify what are the most important of an array of factors contributing to economic growth. The results of such a survey would allow us to parse out the contribution of efficiency change, in the form of loosening of binding constraints, to economic growth and its relative contribution vis-à-vis technical progress.

The Blue Chip Economic Indicators each month survey America's top business economists and ask them to supply their forecasts of U.S. economic growth, inflation, and interest rates, among other business indicators. The survey began in 1976. The experts who make up the Blue-Chip panel are on the order of 50 or so economists and come from a cross section of manufacturing and financial services firms. The Blue Chip Economic Indicators are used by business journalists and by forecasting companies such as the Wall Street Journal, Forbes, and Reuters. The specific information contained in the survey contains forecasts for this year and next from each panel member as well as an average, or consensus, of their forecasts for the following measures of economic activity: Real GDP, GDP price index, Nominal GDP, Consumer price index, Industrial production, Real disposable personal income, Real personal consumption expenditures, Real non-residential fixed investment, Pre-tax corporate profits, 3-mo. Treasury bill rate, 10-yr. Treasury note yield, Unemployment rate, Total housing starts, Auto and light truck sales, Real Net exports. Along with forecasts by each member of the panel is published the consensus forecast for each variable, as well as averages of the 10 highest and 10 lowest forecasts for each variable; a median forecast to eliminate the effects of extreme forecasts on the consensus; the number of forecasts raised, lowered, or left unchanged from a month ago; and a diffusion index that indicates shifts in sentiment that sometimes occur prior to changes in the consensus forecast.[2] One may question the accuracy of the Blue Chip indicators. However, in recent work by A. Chun (2009), the Blue Chip indicators were found to compare favorably with forecasts from the Diebold-Li (2006) model at short horizon forecasts of short to medium maturity interest rates. Development of a survey-based method to decompose total factor productivity growth in a technical change and a technical efficiency change component is motivated not only by an interest in sharper forecasts but also on the possibility that our econometrically based estimates may not be reliable or meaningful. A survey-based set of indicators of such a decomposition may well be all that we can hope for.

How might a questionnaire be constructed? What might be the best survey methods to use in order to solicit answers to such basic questions as:

"Total factor productivity growth if the percentage change in production not attributable to changes in labor, capital, and other inputs. Historically total factor productivity growth in the U. S. has averaged 2%/year. Assuming that all contributions to total factor productivity growth much sum to 1 in percentage terms please answer the following 5 questions-

1. What portion (%) of total factor productivity growth (regress) is due to the innovation provided by new technology?

2. What portion (%) of total factor productivity growth (regress) is due to the better use of existing technology?

3. What portion (%) of total factor productivity growth (regress) is due to changes in government regulations, business climate, or other institutional factors such as political stability and the democratic process?

4. What portion (%) of total factor productivity growth (regress) is due to changes in the scale of operation?

5. What portion (%) of total factor productivity growth (regress) is due to other factors? (Please list them and their relative importance.)"

We expect that information of this sort, collected by a set of experts in countries of the world, will allow us to better understand the role of technology transfer, government regulation, institutional factors such as political stability and the democratic process, and market concentration on the engine for long term and sustainable economic growth: *Total Factor Productivity* growth.

[2] See Wolters Kluwer's Aspen Press website for the Blue Chip Economic Indicators publication by Randell E. Moore:
http://www.aspenpublishers.com/product.asp?catalog_name=
Aspen&product_id=SS01934600&cookie%5Ftest=1).

4 Measuring Technical Change and Efficiency Change Decompositions of Productivity Growth Decompositions

4.1 Index Number Procedures

Either index number or regression based approaches require panel data (at a minimum). The index number approach (Färe et al., 1992) begins by assuming a panel of firms (or countries, etc.) with $i = 1, ..., N$ firms, $t = 1, ..., T$ periods, $j = 1, ..., J$ inputs and $k = 1, ..., K$ outputs. Thus, x_{jit} is the level of input j used by firm i in period t and y_{kit} is the level of output k produced by firm i in period t. Assume an intertemporal production set where input and output observations from all time periods are used. The production technology, $\mathbf{S}$, is

$$\mathbf{S} = \{(x, y) \mid x \in \mathbb{R}_{+}^{J},\ y \in \mathbb{R}_{+}^{K},\ (x, y) \ \text{ is } \ \text{feasible}\}. \tag{5}$$

The efficiency scores are the distances from the frontier. An output-based distance function (Shephard, 1970) , OD, is defined as

$$\mathrm{OD}(x, y) = \min\{\lambda \mid (x, y/\lambda) \in \mathbf{S}\}. \tag{6}$$

Holding the input vector constant, this expression expands the output vector as much as possible without exceeding the boundaries of $\mathbf{S}$. An output efficient firm has a score of 1 and it is not possible for the firm to increase its output without increasing one or more of its inputs. Conversely, an output inefficient firm has $\mathrm{OD}(x, y) < 1$. The productivity index requires output distance functions calculated between periods. $\mathrm{OD}_t(x_{t+1}, y_{t+1}) = \min\{\lambda|(x_{t+1}, y_{t+1/\lambda}) \in \mathbf{S}_t\}$ has the technology of time t and scales outputs in time $t+1$ such that (x_{t+1}, y_{t+1}) is feasible in period t. The observed input-output combination may not have been possible in time t; the value of this expression can exceed one which would represent technical change. $\mathrm{OD}_{t+1}(x_t, y_t) = \min\{\lambda|(x_t, y_{t/\lambda}) \in \mathbf{S}_{t+1}\}$ has the technology of time $t+1$ and scales outputs in time t such that (x_t, y_t) is feasible in period $t+1$. The final equation can be expressed as:

$$\begin{aligned} M(x_{t+1}, y_{t+1}, x_t, y_t) &= \frac{\mathrm{OD}_{t+1}(x_{t+1}, y_{t+1})}{\mathrm{OD}_t(x_t, y_t)} \times && (7) \\ & \left\{ \frac{\mathrm{OD}_t(x_{t+1}, y_{t+1})}{\mathrm{OD}_{t+1}(x_{t+1}, y_{t+1})} \frac{\mathrm{OD}_t(x_t, y_t)}{\mathrm{OD}_{t+1}(x_t, y_t)} \right\}^{1/2}. && (8) \\ &= E_{t+1} \times A_{t+1}. \end{aligned}$$

This index captures the dynamics of productivity change by incorporating data from two adjacent periods. E_{t+1} reflects changes in relative efficiency. A_{t+1} reflects changes in technology between t and $t+1$. For the index, a value below 1 indicates productivity decline while a value exceeding 1 indicates growth. For the index components, values below 1 signify a performance decline while values above 1 signify an improvement. There may be significant shortcomings of this approach, as noted by Førsund and Hjalmarsson (2009), due to potential vintage capital effects or its lack of any obvious inferential theory (Jeong and Sickles, 2004).

4.2 Regression based approaches

Regression based approaches to decomposing productivity growth into technical change and efficiency change components can be explained using the following generic model. Assume that the multiple output / multiple input technology can be estimated parametrically using the output distance function. Since the output distance function, $OD(Y, X) \leq 1$, specifies the fraction of aggregated output (Y) produced by given aggregated inputs (X), it gives us a radial measure of technical efficiency. For an m-output, n-input production technology, the deterministic output distance function can be approximated by

$$\frac{\Pi_j^m Y_j^{\gamma_j}}{\Pi_k^n X_k^{\beta_k}} \leq 1, \qquad (9)$$

where the γ_j's and the β_k's are weights representing the technology of the firm. If one simply multiplies through by the denominator, approximates the terms using a Young Index, a geometric mean with varying weights (Balk, 2009), and adds a disturbance term v_{it} to take account of general statistical noise, and specify a nonnegative stochastic term u_{it} for the firm specific level of radial technical inefficiency, then a regression based approach to decomposing productivity growth into technical change and efficiency change can be specified. The Cobb-Douglas stochastic distance frontier model can be written as:

$$0 = \sum_j \gamma_j \ln y_{j,it} - \sum_k \beta_k \ln x_{k,it} + v_{it} - u_{it}. \qquad (10)$$

The output distance function is linearly homogeneous in outputs and if one imposes this restriction and then normalizes with respect to one y_i(the last) the following expression (Lovell, et al., 1994) can be derived:

$$-\ln(y_J) = \sum_j \gamma_j \ln \widehat{y}_{j,it} - \sum_k \beta_k \ln x_{k,it} + v_{it} - u_{it}, \qquad (11)$$

where $y_J$is the normalized output and $\widehat{y}_j = y_j/y_J$, $j = 1, ..., J-1$. Let $X_{it}^* = -\ln(x_{k,it})$, $Y_{it}^* = \ln(\widehat{y}_{j,it})$, and $Y_{it} = -\ln(y_J)$. Then the stochastic distance frontier is

$$Y_{it} = X_{it}^{*\prime}\beta + Y_{it}^{*'}\gamma + v_{it} - u_{it}, \; i = 1, ...N, t = 1, ...T. \qquad (12)$$

Letting $\varepsilon_{it} = v_{it} - u_{it}$, $X_{it}' = [\ X_{it}^{*\prime}, Y_{it}^{*'}]$, $\xi = [\ \beta, \gamma\ ]$, we obtain the familiar functional form for a stochastic frontier production model under a classical panel data setting:

$$Y_{it} = X_{it}'\xi + \varepsilon_{it}, \; i = 1, ...N, t = 1, ...T. \qquad (13)$$

This is the generic model vehicle for estimating efficiency change using frontier methods. If we assume that innovations are available to all firms and that firms' specific idiosyncratic errors are due to relative inefficiencies then we can decompose sources of TFP growth by adding either an exogenous or a stochastic time trend (see also, Bia, Kao, and Ng, 2007). The panel stochastic frontier model is quite flexible and robust. Technical efficiency of a particular firm (observation) can be consistently estimated. Estimation of the model and the separation of technical inefficiency from statistical noise and from a common technical change component does not require a set of specific assumptions about the parametric distribution of technical inefficiency (e.g., half-normal) and statistical noise (e.g., normal) and dependency structure. For example, it may be incorrect to assume that inefficiency is independent of the regressors since if a firm knows its level of technical inefficiency, this should affect its input choices. Pitt and Lee (1981) and Schmidt and Sickles (1984) have developed random and fixed effects as well as maximum likelihood based estimators for such panel frontier models. To allow for time varying and cross-sectional specific efficiency change one can use a parametrization chosen in Cornwell, Schmidt, and Sickles (1990). They used a quadratic function of time $u_{it} = W_{it}' u_i = \theta_{i1} + \theta_{i2} t + \theta_{i3} t^2$. Other than a quadratic function of time, u_{it} has been modelled as $u_{it} = \gamma(t)\alpha_i = [1+\exp(bt+ct^2)]^{-1}\alpha_i$ (Kumbhakar, 1990), and $u_{it} = \eta_{it}\alpha_i = \exp[-\eta(t-T)]\alpha_i$ (Battese and Coelli, 1992). Both of these approaches used maximum likelihood estimation (MLE) to estimate efficiency. We now turn to other reduced form approaches for measuring the growth in the key components of TFP : efficiency and innovation.

4.3 Bayesian Treatments for Time Varying Inefficiency

Sickles and Tsionas (2008) consider a model similar to the KSS model with common factors whose number is unknown and whose effects are firm-specific. Bayesian inference techniques organized around MCMC are used to implement the models. The model is

$$y_{it} = x_{it}'\beta + \varphi_i(t) + v_{it}, \quad i = 1, ..., n, \quad t = 1, ..., T, \tag{14}$$

where x_{it} and β are $k \times 1$, and $\varphi_i(t)$ is a unit specific unknown function of time. They assume $v_{it} \overset{IID}{\sim} N(0, \sigma^2)$. The model can be written in the form $y_{it} = x_{it}'\beta + \gamma_{it} + v_{it}$. For the ith individual we have $y_i = X_i\beta + \gamma_i + v_i$, $i = 1, ..., n$. Assuming $\gamma_{i1} \leq ... \leq \gamma_{iT}$, they assume a spline prior of the form

$$p(\beta, \sigma, \gamma) \propto \sigma^{-1} \prod_{i=1}^{n} \exp\left(-\frac{\gamma_i' Q \gamma_i}{2\omega^2}\right) = \sigma^{-1} \exp\left(-\frac{1}{2\omega^2}\gamma'(I_T \otimes Q)\gamma\right), \tag{15}$$

where $Q = D'D$, and D is the $(T-1) \times T$ matrix whose elements are $D_{tt} = 1$, $D_{t-1,t} = -1$ and zero otherwise. ω is a smoothness parameter which stands for the degree of smoothness. This prior says that $\gamma_{it} - \gamma_{i,t-1} \sim N\left(0, \omega^2\right)$ or $D\gamma_i \overset{IID}{\sim} N\left(0, \omega^2 I_{T-1}\right)$, that is it assumes that the first derivative of functions $\varphi_i\left(t\right)$ is a smooth function of time. It is possible to allow for smooth second derivatives by using the formulation $\gamma_{it} - 2\gamma_{i,t-1} + \gamma_{i,t-2} \sim N\left(0, \omega^2\right)$, which can be written as $D^{(2)}\gamma_i \overset{IID}{\sim} N\left(0, \omega^2 I_{T-1}\right)$. We can still define $Q = D^{(2)\prime}D^{(2)}$, and the analysis below goes through unmodified. Since γ_{i1} plays the role of an intercept, we can assume $\alpha_i \overset{IID}{\sim} N\left(0, \sigma_\alpha^2\right)$, $i = 1, ..., n$. The model generalizes Koop and Poirier (2004) in the case of panel data with individual-specific intercepts and time effects. Moreover, it does not rely on the conjugate prior formulation for the γ_{it}s which can be undesirable.

The posterior kernel distribution is:

$$
\begin{aligned}
p\left(\beta, \gamma, \sigma | Y, X, \omega\right) \quad &\propto \quad \sigma^{-(nT+1)} \exp\left[-\frac{\left(Y - X\beta - \gamma\right)'\left(Y - X\beta - \gamma\right)}{2\sigma^2}\right] \times \\
&\exp\left[-\frac{1}{2\omega^2}\gamma'\left(I_T \otimes Q\right)\gamma\right], \qquad (16)
\end{aligned}
$$

where $\underset{(nT\times k)}{X} = [X_1', ..., X_n']'$, and $\underset{(nT\times 1)}{Y} = [y_1', ..., y_n']'$. Bayesian inference for this model can be implemented using Gibbs sampling.

4.4 The Latent Class Model

As discussed in Greene (2008), one way to extend the normal-half normal stochastic frontier model (or others) with respect to the distribution of v_i is the finite mixture approach suggested by Tsionas and Greene (2003). This is a class specific stochastic frontier model. The frontier model can be formulated in terms of J 'classes' so that within a particular class,

$$
\begin{aligned}
&f_\varepsilon(\varepsilon_i | class = j) = \tfrac{2}{\sqrt{2\pi(\sigma_u^2 + \sigma_{vj}^2)}}\left[\Phi\left(\tfrac{-\varepsilon_i(\sigma_u/\sigma_{vj})}{\sqrt{\sigma_u^2 + \sigma_{vj}^2}}\right)\right]\exp\left(\tfrac{-\varepsilon_i^2}{2(\sigma_u^2 + \sigma_{vj}^2)}\right), \\
&\varepsilon_i = y_i - \alpha - \beta^{\mathrm{T}} x_i.
\end{aligned} \qquad (17)
$$

Indexation is over classes and involves the variance of the symmetric component of ε_i, $\sigma_{v,j}$. The unconditional model is a probability weighted mixture over the J classes, $f_\varepsilon(\varepsilon_i) = \Sigma_j \pi_j \ \mathrm{f}_\varepsilon(\varepsilon_i |$ class $= j)$, $0 < \pi_j < 1$, $\Sigma_j \pi_j = 1$. Mixing probabilities are additional parameters to be estimated. The model preserves symmetry of the two-sided error component, but provides a degree of flexibility that is somewhat greater than the simpler half normal model. The mixture of normals is, with a finite number of classes, nonnormal. This model can be estimated by Bayesian (Tsionas and Greene, 2003) or classical (Orea and Kumbhakar, 2004; Tsionas and Greene, 2003; Greene, 2004a, 2005) estimation methods. After estimation, a conditional (posterior) estimate of the class that applies to a particular observation can be deduced using Bayes theorem, i.e.:

$$\text{Prob}[class = j|y_i] = \frac{f(y_i|class = j)\text{Prob}[class = j]}{\sum_{j=1}^{J} f(y_i|class = j)\text{Prob}[class = j]} = \hat{\pi}_{j|i}. \quad (18)$$

Individual observations are assigned to the most likely class. Efficiency estimation is based on the respective class for each observation.

Orea and Kumbhakar (2004), Tsionas and Greene (2003) and Greene (2004a, 2005) have extended this model in two directions. First, they allow the entire frontier model, not just the variance of the symmetric error term, to vary across classes. This represents a discrete change in the interpretation of the model. The mixture model is essentially a way to generalize the distribution of one of the two error components. For the fully mixed models, the formulation is interpreted as representing a latent regime classification. The second extension is to allow heterogeneity in the mixing probabilities;

$$\pi_{ij} = \frac{\exp(\theta_j^{\mathrm{T}} z_i)}{\Sigma_{j=1}^{J} \exp(\theta_j^{\mathrm{T}} z_i)}, \qquad \theta_J = 0. \quad (19)$$

The rest of the model is a class specific stochastic frontier model

$$f_\varepsilon(\varepsilon_i|class = j) = \frac{2}{\sigma_j}\phi\left(\frac{\varepsilon_i|j}{\sigma_j}\right)\left[\Phi\left(\frac{-\lambda_j\varepsilon_i|j}{\sigma_j}\right)\right], \quad (20)$$

where $\varepsilon_i|j = y_i - \alpha_j - \beta_j^{\mathrm{T}} x_i$. This form of the model has all parameters varying by class. By suitable equality restrictions, subsets of the coefficients, such as the technology parameters, α and β, can be made generic.

4.5 The Semiparametric Model and Estimators of Technical Efficiency: The Park,Sickles, and Simar SPE Estimators

The models for which the SPE estimators have been derived vary on how the basic model assumptions have been modified to accommodate a particular issue of misspecification of the underlying efficiency model. A number of SPE estimators that differ on the basis of assumed orthogonality of effects and regressors, temporal variation in the efficiency effects, and correlation structure of the population disturbance have been considered and developed in a series of papers by Park and Simar (1994) and Park, Sickles, and Simar (1998, 2003, 2007). For example, when one believes that the effects and all of the regressors are dependent and are unwilling to specify a parametric distribution for the dependency structure then one can specify the joint distribution $h(\cdot,\cdot)$ using kernel smoothers. The Park, Sickles, and Simar (PSS) estimators are based on the theory of semiparametric efficient bounds estimators and utilize an orthogonalization of the scores of the likelihood function with respect to the parameters of interest and the nuisance parameters. The PSS estimators are also adaptive in the terminology of semi-nonparametric estimation theory.

4.6 Alternatives to the Semiparametric Efficient Estimators

There are a number of panel frontier estimators that have been used widely in the empirical efficiency literature. They differ from the SPE estimators based largely on assumptions made about the distribution of the unobserved efficiency effects and about the correlation of efficiency effects and regressors. In order to measure time variant heterogeneity, α_{it} can be specified as:

$$\alpha_{it} = c_{i1}g_{1t} + c_{i2}g_{2t} + \cdots + c_{iL}g_{Lt}, \quad (21)$$

where c_{ir} are unknown parameters, and the *basis functions* g_{ir} are smooth, real-valued functions of x_{it}.This approach is more general than fitting polynomials and can be used to parsimoniously model virtually any temporal pattern of firm efficiency. The firm efficiencies are obtained from the structures of the g_{ir} and from the distribution of the effects α_i. The fixed and random effect models are nested in the mixed efficiency effects specification as are the CSS and SS estimators. Methods for estimating c_{ir}, g_{ir}, and L can be found in Kneip, Sickles, and Song (2009).

4.7 Using Factor Models to Estimate the Solow Residual

The literature on factor models and state-space representations of latent factors using the Kalman filter is quite lengthy and dense. First, we will give a very brief introduction to the factor models. Next we will try to provide some overview of the most recent papers. Then we will select a particular factor model introduced by Kneip, Sickles, and Song to decompose the Solow residual into a technical change and an efficiency change component. Breitung and Eickmeier (2005) provide a very review of factor models and we relay on it in what is discussed below.

4.7.1 Strict Factor Model

Strict factor models are the most simple of the factor model class and utilize the following basic assumptions:

$$\begin{aligned} y_{it} &= \lambda_{i1}f_{1t} + ... + \lambda_{ir}f_{rt} + u_{it} \quad (22) \\ &= \lambda_i' f_t + u_{it} \quad (23) \end{aligned}$$

or

$$\begin{aligned} y_t &= \Lambda f_t + u_t \quad (24) \\ Y &= F\Lambda' + U \quad (25) \end{aligned}$$

where $\Lambda = [\lambda_1 \ \lambda_2 \ ...\lambda_N]'$, $Y = [y_1 \ y_2 \ ... \ y_T]'$, $F = [f_1 \ f_2 \ ... \ f_T]'$, and $U = [u_1 \ u_2 \ ... \ u_T]'$.

For the strict factor models it is usually assumed that u_t are mutually uncorrelated with $E[u_t] = 0$ and $E[u_t u_t'] = \Sigma = diag(\sigma_1^2, \sigma_2^2, ..., \sigma_N^2)$. Moreover, $E[f_t] = 0$. The principle components estimator, the most widely used of the various strict factor specifications, will be inconsistent for fixed N and $T \to \infty$ unless $\Sigma = \sigma^2 I$ as can be seen by considering the principle components estimator as an IV estimator.

4.7.2 Approximate Factor Models

When we allow for $N \to \infty$ we can avoid the restrictive assumptions of strict factor models (Chamberlain and Rothshield, 1983; Stock and Watson, 2002; Bai, 2003) and in this case it is possible to allow for (weak) serial correlation for the idiosyncratic errors. However, persistent and non-ergodic processes are generally ruled out. Idiosyncratic errors can be allowed to be (weakly) cross-correlated and heteroscedastic and (weak) correlation among the factors and the idiosyncratic components are possible. With these and some other technical assumptions Bai (2003) establishes the consistency and asymptotic normality of the principle components estimator for Λ and f_t. However, as noted by Bai and Ng (2005), the small sample properties of this estimator may be severely affected whenever the data is cross-correlated.

4.7.3 Dynamic Factor Models

The dynamic model is given by:

$$y_t = \Lambda_0 g_t + \Lambda_1 g_{t-1} + ... + \Lambda_m g_{t-m} + u_t, \tag{26}$$

where Λ_j are $N \times r$ matrices and g_t is a vector of q stationary factors. Idiosyncratic components of u_t are assumed to be independent (or weakly dependent) stationary processes. Forni, Giannone, Lippi and Reichlin (2004) provide a method to estimate this model. Let $\eta_t = g_t - E[g_t | g_{t-1}, g_{t-2}, ...]$, $f_t = [g_t, g_{t-1}, ..., g_{t-m}]'$ (which is $r = (m+1) \times q$), and $\Lambda = [\Lambda_0, \Lambda_1, ..., \Lambda_m]$. In their first stage the usual principal components are estimated. Note that rather than f_t a rotated version of it, Qf_t, is estimated. The second step estimates a vector-autoregression (VAR) model given by:

$$\hat{f}_t = A_1 \hat{f}_{t-1} + A_2 \hat{f}_{t-2} + ... + A_p \hat{f}_{t-p} + e_t. \tag{27}$$

Note that the rank of the covariance matrix for the e_t term is q since $\hat{f}_t$ includes estimation of lagged factors. If we let $\hat{W}_r$ be the matrix generated by the q largest eigen values of the covariance matrix of e_t, $\hat{\Sigma}_e = \frac{1}{T} \sum_{t=p+1}^{T} e_t e_t'$,then $\hat{\eta}_t = \hat{W}_r' \hat{e}_t$.

An important problem is to determine the number of factors. Forni, Giannone, Lippi, Reichlin (2004) provide an informal criterion based on the proportion of explained variances. Bai and Ng (2005) and Stock and Watson (2005) suggest consistent selection procedures based on principal components. Also, information criteria and tests of the number of factors are suggested by Breitung and Kretschmer (2005). Pesaran (2006) is an interesting paper since it has potential for productivity analysis, in particular frontier production. His paper deals with estimation and inference in panel data models with a general multifactor error structure. The unobserved factors and the individual-specific errors are allowed to follow arbitrary stationary processes and the number of unobserved factors need not be estimated. Individual-specific regressors are filtered with cross-section averages and when the cross-section dimension (N) tends to infinity, the differential effects of unobserved common factors are eliminated.

Carriero, Kapetanios, and Marcellino (2008) look at the forecasting performances of factor models, large scale Bayesian VARs, and multivariate boosting, while Marcellino and Schumacher (2007) focus on factor models that can handle unbalanced datasets in their analysis of the German economy. The approach followed by Doz et al. (2006) and Kapetanios and Marcellino (2006) casts the large factor model in state-space form. Kapetanios and Marcellino (2006) estimate the factors using subspace algorithms, while Doz et al. (2006) exploit the Kalman filter and kernel smoothers. We will focus below on a recent contribution by Kneip, Sickles, and Song (2009) who develop the asymptotic theory for general factor models using a combination of principal components and smoothing spines. In that model not only are methods developed to select the number of factors but also address the potential for nonstationarity. The nonstationarity applies here in regard to a stochastic trend in the standard production function. Below we use the Kneip, Sickles, and Song approach to provide a method to decompose total factor productivity change into a technical change and a technical efficiency change component.

5 The Kneip, Sickles, and Song factor Model Estimator

The Kneip, Sickles, and Song (KSS, 2009) model specifies the factors in the following fashion:

$$Y_{it} = \beta_0(t) + \sum_{j=1}^{p} \beta_j X_{itj} + v_i(t) + \epsilon_{it}, \quad i = 1, \ldots, n, t = 1, \ldots, T, \qquad (28)$$

where denotes a general average function, and $v_i(t)$ are non-constant individual effects. In the context of the production decomposition we consider here think of $\beta_0(t)$ as an exogenous or stochastic long term trend due to technical change in production (Y_{it}) and the $v_i(t)$ as the firm technical efficiency terms in a stochastic frontier production function. Details of the estimator are given in KSS. $\beta_0(t)$ can be eliminated by using centered variables $Y_{it} - \bar{Y}_t$, $X_{ijt} - \bar{X}_{tj}$, where $\bar{Y}_t = \frac{1}{n}\sum_i Y_{it}$ and $\bar{X}_{tj} = \frac{1}{n}\sum_i X_{itj}$ and can be viewed as a nuisance parameter, although in the context of production analysis we will use it to identify the common technical change factor, common to all firms. This is just the diffused technical change that is appropriated by each firm in the industry.

With this normalization, we can write the model as:

$$Y_{it} - \bar{Y}_t = \sum_{j=1}^{p} \beta_j (X_{itj} - \bar{X}_{tj}) + v_i(t) + \epsilon_{it} - \bar{\epsilon}_i, \quad i = 1, \ldots, n, t = 1, \ldots, T, \tag{29}$$

with $\bar{\epsilon}_t = \frac{1}{n}\sum_i \epsilon_{it}$. Identifiability requires that all variables X_{itj}, $j = 1, \ldots, p$ possess a considerable variation over t.

Our focus lies on analyzing $v_i(t)$, $t = 1, \ldots, T$ which of course is motivated by our application in the field of stochastic frontier analysis wherein individual effects determine technical efficiencies and are the main quantity of interest. The coefficients β as well as the functions v_i can be estimated by semiparametric techniques using partial spline estimation where the basic underlying assumption is that $v_i(t)$ represent "smooth" time trends. KSS generalize the usual concept of smoothness by relying on second order differences which also allows them to deal with stochastic processes, for example, random walks. They assume the functions v_i can be represented as a weighted average of an unknown number $L \in \{1, 2, \ldots\}$ of basis functions (common factors) $g_1, \ldots, g_L$ given by

$$v_i(t) = \sum_{r=1}^{L} \theta_{ir} g_r(t), \tag{30}$$

with unknown factor loadings θ_{ir},in which case the centered model can be rewritten

$$Y_{it} - \bar{Y}_t = \sum_{j=1}^{p} \beta_j (X_{itj} - \bar{X}_{tj}) + \sum_{r=1}^{L} \theta_{ir} g_r(t) + \epsilon_{it} - \bar{\epsilon}_t, \quad i = 1, \ldots, n, t = 1, \ldots, T. \tag{31}$$

Parametric mixed effects models of this form are widely used in applications and assume that individual effects can be modeled by linear combinations of *pre-specified* basis function (e.g. polynomials). Cornwell, Schmidt, and Sickles (1990) assume that the v_i can be modeled by quadratic polynomials which in our notation corresponds to an $L = 3$ and g_1, g_2, g_3 forming a polynomial basis. Battese and Coelli (1992) propose a model with $L = 1$ and $g_1(t) = \exp(-\eta(t - T))$ for some $\eta \in \mathbb{R}$. The underlying qualitative assumption is that there exist some common structure characterizing all $v_1, \ldots, v_n$ and that (30) is always fulfilled if the empirical covariance matrix $\Sigma_{n,T}$ of the vectors $(v_i(1), \ldots, v_i(T))'$, $i = 1, \ldots, n$, possesses rank L. This is the setup of factor models considered by Bai (2003, 2005) and Ahn et al. (2005) although the focus of KSS is to analyze non-stationary but smooth time trends. We will outline the basic steps in the estimation process. *Estimation* is based on the fact that under the above normalization $g_1, g_2, \ldots$ are to be obtained as (functional) principal components of the sample $v_1 = (v_1(1), \ldots, v_1(T))', \ldots, v_n = (v_n(1), \ldots, v_n(T))'$. If we let $\Sigma_{n,T} = \frac{1}{n}\sum_i v_i v_i{}'$denote the empirical covariance matrix of $v_1, \ldots, v_n$ (recall that $\sum_i v_i = 0$) and use $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_T$ as well as $\gamma_1, \gamma_2, \ldots, \gamma_T$ to denote the resulting eigenvalues and orthonormal eigenvectors of $\Sigma_{n,T}$,then some algebra reveals the following relationships:

$$g_r(t) = \sqrt{T} \cdot \gamma_{rt} \quad \text{for all} \quad r = 1, \ldots, \ t = 1, \ldots, T, \tag{32}$$

$$\theta_{ir} = \frac{1}{T} \sum_t v_i(t) g_r(t) \quad \text{for all} \quad r = 1, 2, \ldots, \ i = 1, \ldots, n, \tag{33}$$

and

$$\lambda_r = \frac{T}{n} \sum_i \theta_{ir}^2 \quad \text{for all} \quad r = 1, 2, ... \quad . \tag{34}$$

Furthermore, for all $l = 1, 2, \ldots,$

$$\sum_{r=l+1}^{T} \lambda_r = \sum_{i,t} (v_i(t) - \sum_{r=1}^{l} \theta_{ir} g_r(t))^2 = \min_{\tilde{g}_1, \ldots, \tilde{g}_l} \sum_i \min_{\vartheta_{i1}, \ldots, \vartheta_{il}} \sum_t (v_i(t) - \sum_{r=1}^{l} \vartheta_{ir} \tilde{g}_r(t))^2 \tag{35}$$

The estimation algorithm can be represented in five basic steps.

Step 1: Determine estimates $\hat{\beta}_1, \ldots, \hat{\beta}_p$ and functional approximations $\hat{\nu}_1, \ldots, \hat{\nu}_n$ by minimizing

$$\sum_i \frac{1}{T} \sum_t \left(Y_{it} - \bar{Y}_t - \sum_{j=1}^{p} \beta_j (X_{itj} - \bar{X}_{tj}) - \nu_i(t) \right)^2 + \sum_i \kappa \frac{1}{T} \int_1^T (\nu_i^{(m)}(s))^2 ds \tag{36}$$

over all possible values of β and all m-times continuously differentiable functions $\nu_1, \ldots, \nu_n$ on $[1, T]$. Here $\kappa > 0$ is a preselected smoothing parameter and $\nu_i^{(m)}$ denotes the m-th derivative of ν_i.

Step 2: Determine the empirical covariance matrix $\hat{\Sigma}_{n,T}$ of $\hat{v}_1 = (\hat{v}_1(1), \hat{v}_1(2), \ldots, \hat{v}_1(T))', \ldots, \hat{v}_n = (\hat{v}_n(1), \hat{v}_n(2), \ldots, \hat{v}_n(T))'$ by

$$\hat{\Sigma}_{n,T} = \frac{1}{n} \sum_i \hat{v}_i \hat{v}_i'$$

and calculate its eigenvalues $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \ldots \hat{\lambda}_T$ and the corresponding eigenvectors $\hat{\gamma}_1, \hat{\gamma}_2, \ldots, \hat{\gamma}_T$.

Step 3: Set $\hat{g}_r(t) = \sqrt{T} \cdot \hat{\gamma}_{rt}$, $r = 1, 2, \ldots, L$, $t = 1, \ldots, T$, and for all $i = 1, \ldots, n$ determine $\hat{\theta}_{1i}, \ldots, \hat{\theta}_{Li}$ by minimizing

$$\sum_t \left(Y_{it} - \bar{Y}_t - \sum_{j=1}^{p} \hat{\beta}_j (X_{itj} - \bar{X}_{tj}) - \sum_{r=1}^{L} \vartheta_{ri} \hat{g}_r(t) \right)^2 \tag{37}$$

with respect to $\vartheta_{1i}, \ldots, \vartheta_{Li}$.

KSS develop the asymptotic theory underlying this particular factor model. Their main assumption is their **Assumption 5:** *The error terms ϵ_{it} are i.i.d. with $\mathbf{E}(\epsilon_{it}) = 0$, $var(\epsilon_{it}) = \sigma^2 > 0$, and $\mathbf{E}(\epsilon_{it}^8) < \infty$. Moreover, ϵ_{it} is independent from $v_i(s)$ and $X_{is,j}$ for all t, s, j.* They analyze the asymptotic behavior of the parameters of their factor model as $n, T \to \infty$. They do not impose any condition on the magnitude of the quotient T/n and they allow the smoothing parameter κ remain fixed or increase with n, T .

We consider below a range of stochastic frontier productivity models in a series of Monte Carlo experiments based on the panel data model (28):

$$Y_{it} = \beta_0(t) + \sum_{j=1}^{p} \beta_j X_{itj} + v_i(t) + \epsilon_{it}. \tag{38}$$

Two of the existing time-varying individual effects estimators are the random effects GLS (Cornwell, Schmidt, and Sickles (CSS), 1990) and MLE (Battese and Coelli (BC), 1992)]. We also compare the fixed and the random effects estimators (Schmidt and Sickles, 1984). These estimators have been used extensively in the productivity literature that interprets time varying firm effects (time trends) as technical efficiencies. The CSS estimator allows for an arbitrary polynomial in time (usually truncated at powers larger than two) with different parameters for each firm. The BC estimator is a likelihood based estimator wherein the likelihood function is derived from a mixture of normal noise and an independent one-sided efficiency error, usually specified as a half-normal. In the BC estimator, efficiency levels are allowed to differ across firms but the temporal pattern of efficiency is the same for all firms.

We simulate samples of size $n \in \{30, 100, 300\}$ with $T \in \{12, 30\}$ in a model with $p = 2$ regressors and with $\beta_0(t) = 0$ and compare the finite sample performance of four different stochastic frontier estimators. The error process ϵ_{it} is drawn randomly from i.i.d. $\mathbf{N}(0, 1)$. The values of true β are set equal to $(0.5, 0.5)$. In each Monte Carlo sample, the regressors are generated according to a bivariate VAR model as in Park, Sickles, and Simar (2003, 2007):

$$X_{it} = RX_{i,t-1} + \eta_{it}, \text{ where } \eta_{it} \sim \mathbf{N}(0, I_2), \tag{39}$$

and

$$R = \begin{pmatrix} 0.4 & 0.05 \\ 0.05 & 0.4 \end{pmatrix}.$$

To initialize the simulation, we choose $X_{i1} \sim \mathbf{N}(0, (I_2 - R^2)^{-1})$ and generate the samples using (39) for $t \geq 2$. Then, the obtained values of X_{it} are shifted around three different means to obtain three balanced groups of firms from small to large. We fix each group at $\mu_1 = (5, 5)'$, $\mu_2 = (7.5, 7.5)'$, and $\mu_3 = (10, 10)'$. The idea is to generate a reasonable cloud of points for X.

We generate time-varying individual effects in the following ways:

$$\begin{aligned} \text{DGP1} &: \quad v_{it} = \theta_{i0} + \theta_{i1} t + \theta_{i2} t^2 \\ \text{DGP2} &: \quad v_{it} = -\exp(-\eta(t - T)) u_i \\ \text{DGP3} &: \quad v_{it} = v_{i1} g_{1t} + v_{i2} g_{2t} \\ \text{DGP4} &: \quad v_{it} = -u_i \end{aligned}$$

where θ_{ij} $(j = 0, 1, 2) \sim \mathbf{N}(0, 1)/10^2$, $\eta = 0.15$, $u_i \sim$ i.i.d. $|\mathbf{N}(0, 1)|$, v_{ij} $(j = 1, 2) \sim \mathbf{N}(0, 1)$, $g_{1t} = \sin(\pi t/4)$ and $g_{2t} = \cos(\pi t/4)$. DGP1 is the GLS version but the fixed effects treatment is used in the experiments (CSS). We also consider a limited set of simulations in which the data generating process is a random walk. DGP2 is based on Battese and Coelli (1992). DGP3 is considered here to model effects with large temporal variations. DGP4 is the usual constant effects model. Thus, we may consider DGP3 and DGP4 as two extreme cases among the possible functional forms of time-varying individual effects.

For the KSS estimator, cubic smoothing splines were used to approximate v_{it} in Step 1, and the smoothing parameter κ was selected by using generalized cross-validation.[3] Most simulation experiments were repeated 1,000 times except the cases for $n = 300$ for which 500 replications were carried out. To measure the performances of the effect and efficiency estimators, we used normalized mean squared error (MSE):

$$R(\widehat{v}, v) = \frac{\sum_{i,t} (\widehat{v}_{it} - v_{it})^2}{\sum_{i,t} v_{it}^2}.$$

For the estimates of technical efficiency, we also considered the Spearman rank order correlations of true average technical efficiency across the simulations and the estimates of technical efficiency based on the different estimators.

Before we present the simulation results, we briefly introduce the other estimators. For the Within and generalized least squares (GLS) estimators which treat the effects as temporally varying, once individual effects v_{it} are estimated, technical efficiency is calculated as $TE = \exp\{v_i - \max(v_i)\}$ following Schmidt and Sickles (1984). Battese and Coelli (1992) employ the maximum likelihood estimation method to estimate the following equation:

$$Y_{it} = \beta_0 + \sum_{j=1}^{p} \beta_j X_{itj} + \epsilon_{it} - u_{it}, \tag{40}$$

where the time-varying effects terms are defined as $u_{it} = \eta_{it} u_i = \{\exp[-\eta(t - T)]\}\, u_i$ for $i = 1, \ldots, n$. Technical efficiency is then calculated as $TE_{BC} = \exp(-u_{it})$. Cornwell, Schmidt, and Sickles (1990) approximate time-varying effects by a quadratic function of time. The model can be written as

$$Y_{it} = X_{it}'\beta + W_{it}'\delta_i + \varepsilon_{it}, \tag{41}$$

where $W_{it} = [1, t, t^2]$. If W contains just a constant term then the model reduces to the standard panel data model with heterogeneity in the intercept. If we let $\delta_i = \delta_0 + u_i$ then the model can be rewritten as

$$\begin{aligned} Y_{it} &= X_{it}'\beta + W_{it}'\delta_0 + \omega_{it}, & (42)\\ \omega_{it} &= W_{it}'u_i + \varepsilon_{it} = v_i(t) + \epsilon_{it}, & (43) \end{aligned}$$

[3] We let $\kappa = (1-p)/p$ and chose p among a selected grid of 9 equally spaced values between 0.1 and 0.9 so that generalized cross-validation rule is minimized.

or

$$Y = X\beta + W\delta_0 + \omega, \quad (44)$$

$$\omega = Qu + \varepsilon = v + \varepsilon. \quad (45)$$

The Within estimator for β is then

$$\widehat{\beta}_{cssw} = (X'M_Q X)^{-1} X'M_Q Y,$$

where $M_Q = I - Q(Q'Q)^{-1}Q'$, $Q = \text{diag}(W_i)$, $i = 1, \ldots, n,$. Technical efficiency is defined as $TE_{CSS} = \exp\{v_{it} - \max(v_{it})\}$. For the KSS estimator, technical efficiency is calculated similarly as for the CSS estimator.

We now we present the simulation results. Tables 1-4 present mean squared errors (MSE) of coefficients, effects, and efficiencies, and the Spearman rank order correlation coefficient of efficiencies for each DGP. Also, average optimal dimensions, L, chosen by $C(l)$ criterion are reported in the last column of second panel in each Table. Note first that optimal dimension, L, is correctly chosen for the KSS estimator in all DGPs[4] Thus, we can verify the validity of the dimension test $C(l)$ discussed in Section 2.

For DGP1, the performances of the KSS estimator are better than the other estimators by any standards. This is true even when the data is as small as $n = 30$ and $T = 12$. In particular, the KSS estimator outperforms the other estimators in terms of MSE of efficiency. Since the data are generated by DGP1, we may expect that CSS estimator performs well. This is true for $T = 30$. However, if T is small ($T = 12$), the CSS estimator is no better than the other estimators. The performances of Within, GLS, and BC estimators generally get worse as T increases. Results in Table 1a, generated from a random walk data generating process, are comparable to those in Table 1.

For DGP2, when data is generated using the model specification of the BC estimator the performances of the KSS estimator is comparable to or sometimes better than that of the BC estimator. The BC estimator seems to work fine for the estimation of effects and efficiencies. In terms of MSE of coefficients, however, it appears that the BC estimator is not reliable when T is large ($T = 30$). The Within and GLS estimators also suffer from substantial distortions when T is large. DGP3 generates effects with large temporal variations. Hence, simple functions of time such as used in the CSS or BC estimators are not sufficient for this type of DGP. However, the KSS estimator does not impose any specific forms on the temporal pattern of effects, and thus it can approximate any shape of time varying effects. We may then expect good performances of the KSS estimator even in this situation, and results in Table 3 confirm such belief. On the other hand, the other estimators suffer from severe distortions in the estimates of effects and efficiencies, although coefficient estimates look

[4] Although DGP1 consists of three different functions, $[1, t, t^2]$, t^2 term is dominating as T gets large. Thus a one dimensional model is sufficient to approximate the effects generated by DGP1.

reasonably good. In particular, rank correlations of efficiencies are almost zero when T is large.

DGP4 represents the reverse situation so that there is no temporal variation in the effects. Hence, the Within and GLS estimators work very well. Now, our primary question is what are the performances of KSS estimator in this situation. As seen in Table 4, its performances are fairly well and comparable to those of the Within and GLS estimators. Therefore, the KSS estimator may be safely used even when temporal variation is not noticeable.

In summary, simulation experiments show that either if constant effects are assumed when the effects are actually time-variant, or if the temporal patterns of effects are misspecified, parameters as well as effect and efficiency estimates become severely biased. In these cases, large T increases the bias, and large n does not help solve the problem. On the other hand, our new estimator performs very well regardless of the assumption on the temporal pattern of effects, and may therefore be preferred to other existing estimators in these types of empirical settings, among potentially many others.

6 Conclusion

We have discussed the Solow residual and how it has been interpreted and measured in neoclassical production literature and in the complementary literature on productive efficiency. We have also pointed out why panel data are needed to measure productive efficiency and innovation and thus link the two strands of literature. We provided a discussion on the various estimators used in the two literatures, focusing on one in particular, the factor model and evaluated in finite samples the performance of a particular factor model, the KSS model.

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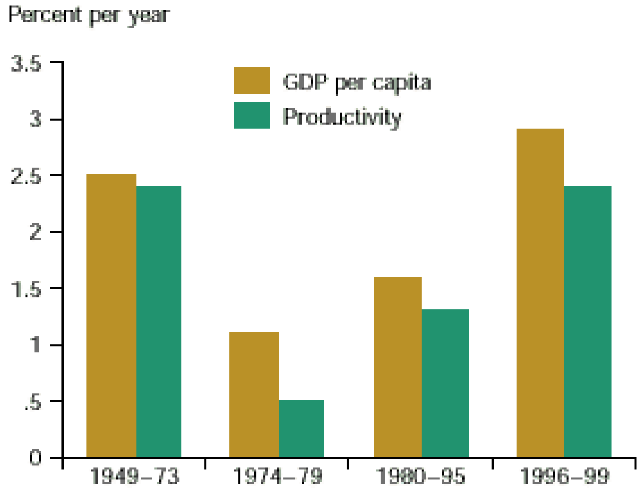
Chart 1
Productivity Growth Has a Big Impact on Living Standards



SOURCES: Department of Commerce; Department of Labor; author's calculations.

Table 1. Monte Carlo Simulation Results for DGP1

MSE of Coefficients*

N	T	Within	GLS	BC	CSS	KSS
30	12	0.9107	0.6039	0.4933	0.8863	0.4998
	30	4.5286	4.0001	1.1767	0.2329	0.1462
100	12	0.2635	0.1438	0.1454	0.2504	0.1170
	30	1.2219	1.0068	1.4172	0.0726	0.0410
300	12	0.0801	0.0402	0.0360	0.0790	0.0343
	30	0.3409	0.2848	0.1456	0.0258	0.0151

MSE of Effects

N	T	Within	GLS	CSS	KSS	L
30	12	0.6159	0.5692	0.4675	0.2278	1.1200
	30	0.4476	0.4455	0.0051	0.0037	1.0510
100	12	0.5940	0.5755	0.4438	0.1769	1.0620
	30	0.4539	0.4531	0.0050	0.0100	1.0590
300	12	0.6068	0.5990	0.5504	0.1964	1.0341
	30	0.4379	0.4376	0.0064	0.0025	1.0500

MSE of Efficiencies

N	T	Within	GLS	BC	CSS	KSS
30	12	0.3429	0.3255	0.1485	0.3329	0.0921
	30	0.6967	0.7005	0.8430	0.2069	0.0289
100	12	0.4415	0.4294	0.3817	0.3969	0.0529
	30	0.8305	0.8279	1.1184	0.2790	0.0236
300	12	0.5102	0.5070	0.4574	0.4575	0.0364
	30	0.9401	0.9400	1.6111	0.3470	0.0154

Spearman Rank Correlation of Efficiencies

N	T	Within	GLS	BC	CSS	KSS
30	12	0.5052	0.5004	0.8085	0.7692	0.9806
	30	0.4829	0.4834	0.7533	0.9841	0.9980
100	12	0.3886	0.3886	0.5656	0.7837	0.9923
	30	0.3885	0.3885	0.5900	0.9871	0.9993
300	12	0.3037	0.3037	0.6267	0.7771	0.9924
	30	0.2805	0.2805	0.5469	0.9878	0.9995

Note: * is multiplied by 10^2.

Table 2. Monte Carlo Simulation Results for DGP2

MSE of Coefficients*

N	T	Within	GLS	BC	CSS	KSS
30	12	2.2939	1.6274	0.3427	0.8901	0.4661
	30	161.0314	106.1230	9.6053	5.4253	0.1499
100	12	0.7709	0.6094	0.1149	0.2505	0.1206
	30	53.4336	39.4729	8.1635	1.9065	0.0403
300	12	0.2873	0.1760	0.0339	0.0800	0.0371
	30	18.4371	11.9706	1.3051	0.6689	0.0141

MSE of Effects

N	T	Within	GLS	CSS	KSS	L
30	12	0.3892	0.3753	0.0699	0.1401	1.0720
	30	0.7443	0.7351	0.0202	0.0705	1.0430
100	12	0.4678	0.4642	0.0701	0.2120	1.0350
	30	0.8029	0.8007	0.0217	0.1024	1.0050
300	12	0.4475	0.4452	0.0617	0.1966	1.0260
	30	0.7911	0.7902	0.0213	0.0986	1.0020

MSE of Efficiencies

N	T	Within	GLS	BC	CSS	KSS
30	12	0.2260	0.1951	0.0321	0.2586	0.0786
	30	0.7924	0.7321	0.0096	0.5236	0.0544
100	12	0.2598	0.2473	0.0400	0.2944	0.0787
	30	0.7361	0.7548	0.0091	0.5788	0.0116
300	12	0.2695	0.2618	0.0338	0.3607	0.0916
	30	0.7542	0.7342	0.0213	0.5568	0.0040

Spearman Rank Correlation of Efficiencies

N	T	Within	GLS	BC	CSS	KSS
30	12	0.8941	0.8914	0.9950	0.9716	0.9976
	30	0.6239	0.6293	0.9993	0.8871	0.9946
100	12	0.8283	0.8249	0.9981	0.9784	0.9966
	30	0.5349	0.5342	0.9997	0.8917	0.9999
300	12	0.8448	0.8446	0.9982	0.9726	0.9938
	30	0.5478	0.5479	0.9982	0.8820	1.0000

Note: * is multiplied by 10^2.

Table 3. Monte Carlo Simulation Results for DGP3

MSE of Coefficients*

N	T	Within	GLS	BC	CSS	KSS
30	12	1.6631	0.6852	0.6986	2.7261	0.7099
	30	0.5340	0.2621	0.2779	0.6766	0.1821
100	12	0.4224	0.1597	0.1649	0.6866	0.1290
	30	0.1468	0.0667	0.0715	0.1853	0.0396
300	12	0.1549	0.0606	0.0638	0.2429	0.0378
	30	0.0516	0.0250	0.0281	0.0649	0.0138

MSE of Effects

N	T	Within	GLS	CSS	KSS	L
30	12	1.0897	1.0259	1.1143	0.2710	2.1609
	30	1.0432	1.0240	1.0840	0.1140	2.0483
100	12	1.0602	1.0393	1.0672	0.2351	2.0585
	30	1.0364	1.0294	1.0829	0.0929	2.0102
300	12	1.0424	1.0353	1.0197	0.2081	2.0061
	30	1.0307	1.0285	1.0734	0.0822	2.0021

MSE of Efficiencies

N	T	Within	GLS	BC	CSS	KSS
30	12	2.1298	2.4086	7.9252	1.4860	0.2583
	30	2.2636	2.5640	5.0451	1.6066	0.1031
100	12	2.4655	2.6934	12.8728	1.4582	0.2175
	30	7.1729	7.6171	18.6293	4.2421	0.1109
300	12	3.8455	3.9679	25.7966	1.9365	0.2085
	30	8.9848	9.2055	26.4074	4.8352	0.1122

Spearman Rank Correlation of Efficiencies

N	T	Within	GLS	BC	CSS	KSS
30	12	0.1754	0.1729	0.0408	0.2535	0.9298
	30	0.0597	0.0600	-0.0181	0.0019	0.9842
100	12	0.2050	0.2051	0.1513	0.2674	0.9277
	30	0.0499	0.0498	0.0477	0.0325	0.9731
300	12	0.2131	0.2130	0.0754	0.2615	0.9236
	30	0.0575	0.0574	0.0136	-0.0248	0.9691

Note: * is multiplied by 10^2.

Table 4. Monte Carlo Simulation Results for DGP4

MSE of Coefficients*

N	T	Within	GLS	BC	CSS	KSS
30	12	0.5732	0.3586	0.3734	0.8634	0.6515
	30	0.2023	0.1513	0.1504	0.2319	0.2292
100	12	0.1741	0.1346	0.1260	0.2529	0.1816
	30	0.0571	0.0537	0.0510	0.0695	0.0596
300	12	0.0609	0.0360	0.0364	0.0910	0.0617
	30	0.0218	0.0164	0.0142	0.0258	0.0221

MSE of Effects

N	T	Within	GLS	CSS	KSS	L
30	12	0.4390	0.3500	1.2061	0.5407	1.0250
	30	0.1681	0.1465	0.4526	0.2217	1.0130
100	12	0.2769	0.2631	0.8046	0.2988	1.0300
	30	0.1082	0.1065	0.3145	0.1186	1.0200
300	12	0.2689	0.2614	0.7959	0.2799	1.0250
	30	0.0969	0.0954	0.2871	0.1015	1.0220

MSE of Efficiencies

N	T	Within	GLS	BC	CSS	KSS
30	12	0.1211	0.0993	0.1178	0.2600	0.1344
	30	0.0488	0.0421	0.0416	0.1205	0.0595
100	12	0.1719	0.1622	0.0478	0.3488	0.1778
	30	0.0798	0.0763	0.0252	0.1857	0.0829
300	12	0.2124	0.2075	0.0449	0.4120	0.2157
	30	0.0914	0.0907	0.0231	0.2168	0.0938

Spearman Rank Correlation of Efficiencies

N	T	Within	GLS	BC	CSS	KSS
30	12	0.9964	0.9742	0.9738	0.9481	0.9955
	30	0.9982	0.9804	0.9787	0.9757	0.9977
100	12	0.9989	0.9883	0.9896	0.9106	0.9987
	30	0.9997	0.9946	0.9949	0.9528	0.9996
300	12	0.9997	0.9997	0.9995	0.8946	0.9996
	30	0.9997	0.9995	0.9997	0.9588	0.9997

Note: * is multiplied by 10^2.